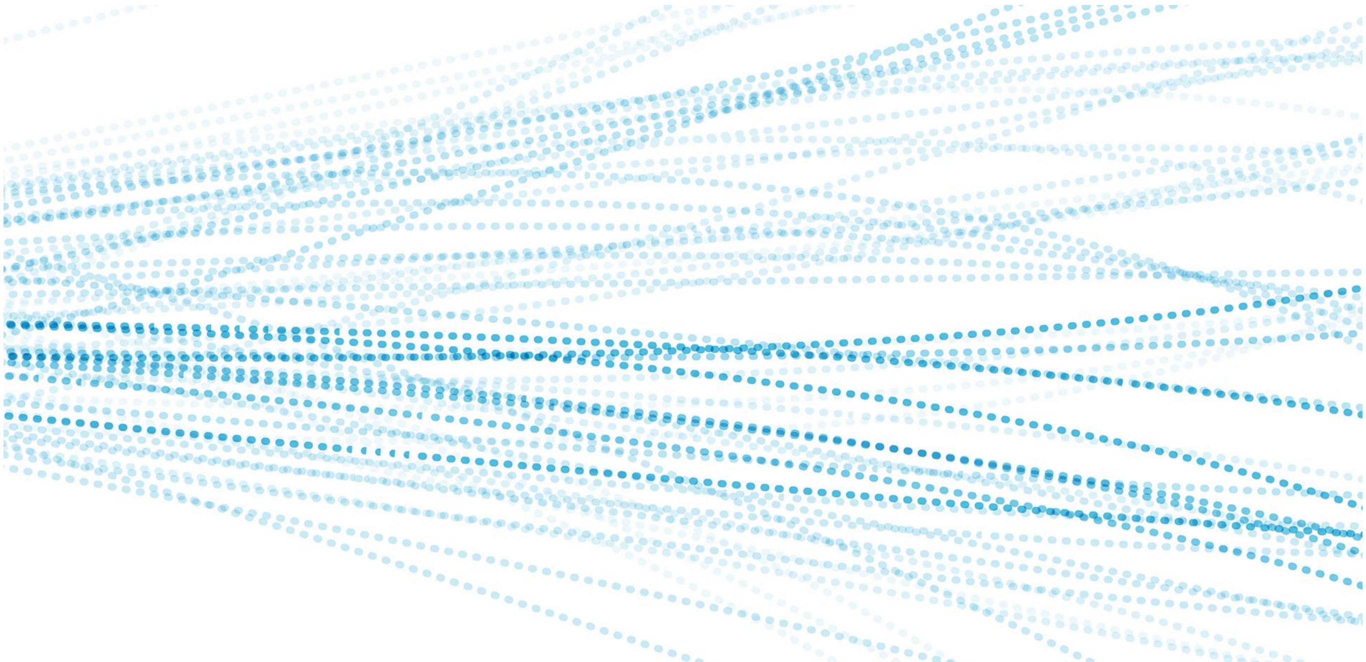

# Introduction to the changes in the Solvency II yield curve and the implications for hedging

## A close examination of the new Alternative Methodology for yield curve construction under Solvency II

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EIOPA, the European Insurance and Occupational Pensions Authority, as part of its Solvency II review, is updating the approach to calculating the yield curve, which reflects a balancing of several different, competing concerns. The existing methodology is based on the Smith-Wilson extrapolation, with a last-liquid point (LLP) grading to an ultimate forward rate (UFR). At present, for the euro, the LLP is 20 years, which does not reflect current liquidity in the underlying market but instead reflects the available technical considerations[1] in the market at the time the Omnibus II Directive came into force.[2]

In October 2009, the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS) issued guidance to the European Commission outlining criteria that any risk-free term structure must satisfy:

1. No credit risk: The rates should be free of credit risk.
2. Realism: It should be possible to earn the rates in practice.
3. Reliability: The determination of the rates should be reliable and robust.
4. High liquidity: The rates should be based on financial instruments from deep, liquid, and transparent markets.
5. No technical bias: The rates should have no technical bias.

[1] Which is to say that it was a combination of technical and political considerations. Twenty years is a compromise between stakeholders, some of whom preferred a 10Y LLP, and some of whom preferred 30Y. However, the 20Y point is also justifiable using residual bond considerations. Consultation Paper on the Opinion on the 2020 review of Solvency II, p. 33-34.

[2] Article 77a requires deep bond, liquid, and transparent bond markets up to the LLP, where derived interest rates rely on swap rates. Recital 30 of the Omnibus II Directive states that one can match liability cash flows up to the LLP with bond cash flows. Furthermore, market conditions similar to those at the date of entry into force of the Omnibus II Directive indicate that the LLP is 20 years.

CEIOPS further noted that any extrapolation methodology should satisfy all of the above as well, with the exception of liquidity. They further noted that volatility estimates of long-term rates may cause undesirable pro-cyclical impacts to liability valuation, and thus, any approach should consider financial stability, too. Hence the balancing between realistic valuation and financial stability.[3]

First, there has been growing concern that both the LLP and the UFR rate were set at levels that no longer accurately reflect the economic value of liabilities, particularly for long-term guarantees. Given the current low-interest-rate environment, the difference between the market rates and 20, 30, or 50 years and the extrapolated rates is significant. This has created supervisory concern, amongst others from the European Systemic Risk Board (ESRB), that the technical provisions are understating long-term liabilities, a clear risk to situations where a transfer of liabilities is necessary.[4]

Second, even absent a transfer, the negative repercussions of such a deficiency to policyholder protection and prudent risk management is also concerning. The current scheduled reduction of the UFR will not resolve this issue on a short notice, since it can only change by at most 15 basis points per annum. Note that the UFR also contains an inflation component set at 2%. This, in addition to the real rate component, which is estimated as a long-term average (from 1961), will delay the realization of a negative UFR for some time.[5] Third, "[w]here the extrapolated risk-free interest rates differ from the market rates, undertakings need to decide whether they hedge the risk as it is reflected in their solvency balance sheet or whether they hedge the risk that actually exists in the financial markets."[6] Since these two different targets can have material implications, where targeting one can result in less favorable results on the other, there is a concern that this creates undesirable risk management incentives. An approach that more closely aligns economics with the regulatory picture will create better alignment.

Finally, ongoing deep, liquid, and transparent assessments of the swap markets in various currencies indicate that there should also be adjustments to many currency LLPs. The above considerations gave rise to five different options for estimating risk-free term structures that were part of the Consultation Paper on the 2020 review:

1. No change.
2. LLP stays at 20 years for the euro and additional safeguards are introduced in pillars 2 and 3.
3. The LLP is increased to 30 years for the euro.
4. The LLP is increased to 50 years.
5. An alternative extrapolation method is adopted.

Of these options, EIOPA has tentatively proposed the adoption of option 5 in the most recent impact assessments,[7] which the remainder of this paper will discuss and evaluate.

# Preliminaries

To ensure comfort with the terms and methods described in this paper, we will first describe a few key concepts. There are a multiple of interest rates currently in use, but they generally fall into the following categories: par rates, spot rates (or zero rates), forward rates, and discount rates.

Let Z(t, T) denote the value of a zero-coupon bond[8] with maturity value 1 purchased at time t with maturity T (and therefore has a time to maturity of T-t). The value of this zero-coupon bond is equivalent to the discount rate for a cash flow occurring at time T, discounted to the valuation time t. The unique interest rate $r_{T-t}$, such that

$$Z(t,T)(1+r_{T-t})^{(T-t)}=1$$

is the annualized spot rate, or zero rate, for the period t to T.

[3] Consultation Paper on the Opinion on the 2020 review of Solvency II, p. 23-24.

[4] See ESRB: Regulatory risk-free yield curve properties and macro prudential consequences, August 2017.

[5] Consultation Paper on the Opinion on the 2020 review of Solvency II, p. 23.

[6] Consultation Paper on the Opinion on the 2020 review of Solvency II, p. 23-24.

[7] The exact quote is: "The consultation paper set out EIOPA's tentative proposals to change Solvency II. In light of the consultation feedback and the data received so far EIOPA further developed its position in order to arrive at the proposals included in this information request. Nevertheless, EIOPA's position as reflected in this information request is not final." Technical specification of the information request on the 2020 review of Solvency II (2020) p. 2.

[8] A zero-coupon bond is a bond that has only one cash flow, which is the repayment of the principal amount at maturity.

Par rates differ from the above spot rates because they reflect a sequence of yields-to-maturity that price each bond at par value (that is, the face value of the bond). Par rates are determinable from spot rates by solving the following equation for each bond:

$$100 = \frac{PMT}{(1+r_1)^1} + \frac{PMT}{(1+r_2)^2} + \frac{PMT}{(1+r_3)^3} + \cdots + \frac{PMT + Principal}{(1+r_N)^N}$$

where 100 reflects the par value of the bond (with face amount 100), PMT are the period coupon payments, and $r_i$ are the spot rates for time i.[9]

"The forward rate at current time t for period $T_1$ to $T_2$, $t \le T_1 \le T_2$, is the rate agreed at time t at which one can [theoretically?] borrow or lend money from $T_1$ to $T_2$."[10] Deriving this forward rate is straightforward using spot rates:

$$f_{T_1,T_2} = \frac{r_{T_2}(T_2 - t) - r_{T_1}(T_1 - t)}{T_2 - T_1},$$

if rates are continuously compounded, and

$$(1 + r_{T_1})^{T_1 - t}(1 + f_{T_1,T_2})^{T_2 - T_1} = (1 + r_{T_2})^{T_2 - t}$$

in the case where rates are annually compounded.

## Limitations of Smith-Wilson

Hagan and West (2006) described criteria that are generally desirable for any interest rate interpolation[11] method. Such methods should:

1. Match market data and be computationally fast to fit
2. Generate smooth forward rates
3. Generate non-negative forward rates
4. Generate stable forward rates—that is, small changes in the values of market inputs should generate only small changes in forward rates
5. Generate local hedges—that is, you should be able to hedge a liability with market instruments with tenors that closely match that of the liability

Not all of these criteria hold true today. Market events have unequivocally shown that rates can and do go negative. We therefore modify criterion 3 to state that discount factor should not be negative. With this change, the above criteria form a suitable basis for evaluating different interpolation approaches.

As the scope of this paper relates to hedging under the new EIOPA yield curve method, it behooves us to address the challenges that exist for insurers hedging using Smith-Wilson. We will only do so at a high level, though, as others have capably handled the technicalities of the subject (see Lagerås & Lindholm, 2016).

From a practical perspective—for example, for hedgers—different problems arise from the Smith-Wilson methodology. Perhaps the most obvious problem with Smith-Wilson is the need to hold oscillating positions in adjacent instruments used to construct the curve.[12] That is, if you are long a bond with maturity t, then the optimal hedge at t+1 will likely be short. The next will be long, and so on and so forth. One can avoid this problem for shorter tenors by using swaps or coupon-paying bonds, but the problem is unavoidable for longer tenors and results in very curious hedge portfolios. Perhaps even more unfortunate is the fact that as the hedged liability ages, rebalancing this hedge portfolio will require rolling each instrument into its opposite position (i.e., a long position today becomes a short position in a year, and vice versa). This implies significant portfolio turnover, and large trading expenses. More details on this are described in the analysis sections later in this paper.

[9] Note that if the bond is a zero-coupon bond, the par rate equals the spot rate.

[10] Stephen Blyth, An Introduction to Quantitative Finance (2014), p. 24.

[11] While Hagan and West (2006) focused on interpolation only, their criteria are equally important for any extrapolation methodology, especially in cases where market data informs the extrapolation and hedging is under consideration for that portion of the curve.

[12] This is an important issue, but others have covered this subject well. We discuss this point in more detail below, but readers interested in a more involved, technical discussion are encouraged to consult Lagerås & Lindholm (2016).

It follows from the above observations that the Smith-Wilson method does not satisfy criteria 4 and 5. Criterion 3 is also not satisfied, even when relaxed to require only that discount rates stay non-negative. EIOPA (2010) actually noted this possibility. This can occur when the par curve has a very steep slope at high tenors, and thus forward rates are very high. While negative forward rates do not give rise to the pathologies long believed to be inevitable, negative discount rates are very much problematic, as that implies that spot rates are imaginary.[13]

That leaves just criteria 1 and 2, which the Smith-Wilson method does satisfy. Unfortunately, that leave substantial challenges to any insurer trying to hedge their liabilities consistent with Solvency II. Next, we will discuss the proposed changes to yield curve construction.

## Introduce the changes proposed by the 2020 Solvency II Review

EIOPA, in its 2020 review of the Solvency II technical provisions, identified a number of important issues that any methodology for estimating the risk-free curve needed to address.[14] At a high level, reliability and robustness of the term structure is an important prerequisite for any regulatory system. This includes balancing the tension between market consistency and muting pro-cyclicality. Additionally, liability valuations were very sensitive to the identification and calculation of the last liquid point (LLP). This is a key issue to address.

Their 2017 report identified four requirements risk-free rates needed to satisfy: “realistic estimate of the time value of money, consistent application, adequate risk management, and limiting pro-cyclicality.” This links to the four issues that are highlighted in the recent Consultation Paper:

1. Underestimation of technical provisions
2. Risk management incentives
3. Stability of the solvency position and impact on financial stability
4. Evidence on DLT assessments

EIOPA considered five different approaches to adjusting the interpolation and extrapolation of the risk-free curves. Option 5, the ‘alternative extrapolation method’, is the focus of this paper. EIOPA describes it as having the following advantages and disadvantages over the current approach:

### Pros

1. Would slightly improve the market consistency of the risk-free interest rate term structure and thereby partially mitigate the risk of underestimation of technical provisions
2. Slightly closer to outcome of the Deep, Liquid, and Transparent (DLT) assessment of euro swap market than current LLP of 20 years
3. Would reduce wrong incentives for risk management but not fully remove them
4. Would be applicable to all currencies, and an exemption for the euro would no longer be required

### Cons

1. Moderate increase of volatility of own funds. There are concerns that the increased volatility could have pro-cyclical effects where insurers do not closely match their hedge to the full yield curve.

The last observation is worth expanding upon. The alternative method, as discussed below, incorporates market information partially between the first smoothing point and the last liquid forward rate. As this part of the curve is not completely market-consistent, there is a colorable argument against measuring liability sensitivities to this part of the derived yield curve.

[13] Lagerås & Lindhold (2016).

[14] Consultation Paper on the Opinion on the 2020 review of Solvency II, p. 22-28.

# The technicalities of the alternative method

In the introduction of the alternative method for deriving the risk-free term structure we distinguish between the liquid part, up to the first-smoothing point (FSP) and the extrapolated part where concepts as the last liquid forward rate (LLFR) and the ultimate forward rate (UFR) are used.

### THE LIQUID PART

The term structure for maturities up to and including the FSP is fully determined by market information and thus plays a similar role as the last liquid point in the current extrapolation method. Therefore, the FSP is—similarly as for the current LLP for the euro—determined using the residual bond criterion. Hence, the FSP is equal to 20 years for the euro. For all other currencies the FSP would also be set according to the residual bond criterion. The rationale for using the residual bond criterion is that it indicates the relative availability of bond cash flows to match liabilities beyond the FSP (or LLP). The matching criterion would no longer play a role when determining the FSP. The FSP would be set equal to the closest maturity for which the reference rates are considered DLT. In order to ensure stability of the FSP maturities, the FSP maturity is only changed if the residual bond criterion delivers results that vary for two consecutive years.

### THE EXTRAPOLATED PART

The term structure for maturities beyond the FSP depends on the LLFR and the UFR. The extrapolation takes place at the level of forward rates.

Forward rates beyond the FSP are a weighted average of the LLFR and the UFR where the weight on the UFR gets larger for longer maturities. The formula for the weighting factor is derived from the Vasicek model for interest rates. It is parametrized with a convergence factor of a = 10%. A larger convergence factor implies that the weight on the UFR gets larger. In this way, the convergence factor also influences the volatility of the extrapolated forward rates.

The LLFR is a weighed combination of forward rates pre- and post the FSP with weights that depend on the liquidity of the respective rates according to the annual EIOPA DLT assessment. In this way, market information beyond the FSP is also partially taken into account, but only as long as the respective swap rates are sufficiently liquid for these maturities and to the extent of the liquidity of the rates.

The LLFR for the euro depends on all DLT swap rates with a maturity up to 50 years, i.e., 15, 20, 25, 30, 40, and 50, where the weight for the 30-year swap rate is significantly larger than for the 40- and 50-year swap rates, as it is more liquid.

# Technical description

Our notation mirrors the EIOPA consultation document for the sake of consistency.

$r_t$ = the (par) rate at maturity t

$z_t$ = the spot zero-coupon rate at maturity t

$f_{t1,t2}$ = the forward rate between maturities t1 and t2

In our description we make a distinction between the liquid part, up to the FSP, and the extrapolated part. A separate section is explaining the role of the volatility adjustment.

### THE LIQUID PART

The zero-coupon rate is derived from the par swap rate by means of bootstrapping, starting with the one-year swap. From $(1 + r_1) / (1 + z_1) = 1$ follows that $z_1 = r_1$. The two-year rate is determined by discounting the cash flows of the two-year swap (only the fixed-income part) against the one-year and two-year rates equaling the discounted value at 1:

$$\frac{r_2}{1+z_1}+\frac{1+r_2}{(1+z_2)^2}=1$$

$$z_2 = \sqrt{\frac{1 + r_2}{1 - \frac{r_2}{1 + z_1}}} - 1$$

$z_3$ to $z_{10}$ are handled analogously.

The forward rates are computed as follows:

$$(1 + z_2) = (1 + z_1)(1 + f_{1,2})$$

$$f_{1,2} = \frac{(1 + z_2)^2}{(1 + z_1)} - 1$$

For the maturities beyond 10 years, less market data is used. For the maturities between the 12, 15, and 20 years, interest rates are interpolated. For example, in order to compute the 16-year zero-coupon rate, an assumption has to be made. Here we assume that the one-year forward rates between 15 and 20 years are the same. Using this assumption, we can write the following:

$$(1 + z_{16})^{16} = (1 + z_{15})^{15}(1 + f_{15,16}) = (1 + z_{15})^{15}(1 + f_{15,20})$$

$$(1 + z_{17})^{17} = (1 + z_{16})^{16}(1 + f_{16,17}) = (1 + z_{15})^{15}(1 + f_{15,20})^2$$

$$(1 + z_{18})^{18} = (1 + z_{17})^{17}(1 + f_{17,18}) = (1 + z_{15})^{15}(1 + f_{15,20})^3$$

$$(1 + z_{19})^{19} = (1 + z_{18})^{18}(1 + f_{18,19}) = (1 + z_{15})^{15}(1 + f_{15,20})^4$$

$$(1 + z_{20})^{20} = (1 + z_{19})^{19}(1 + f_{19,20}) = (1 + z_{15})^{15}(1 + f_{15,20})^5$$

Based on this we can write the following for the cash value of a 20-year swap:

$$\frac{r_{20}}{1 + z_1} + \frac{r_{20}}{(1 + z_2)^2} + \cdots + \frac{r_{20}}{(1 + z_{19})^{19}} + \frac{1 + r_{20}}{(1 + z_{20})^{20}}$$

$$r_{20} \sum_{t=1}^{15} \frac{1}{(1 + z_t)^t} + \frac{1}{(1 + z_{15})^{15}} \sum_{t=1}^{5} \frac{1}{(1 + f_{15,20})^t} + \frac{1}{(1 + z_{15})^{15}(1 + f_{15,20})^5} = 1$$

Using a numeric procedure $f_{15,20}$ can be determined and then replaced in the equation above in order to determine $z_{16}$ to $z_{20}$. The same procedure is used to determine all over one-year forward rates.

Zero-coupon yields up to and including the FSP are equal to zero-coupon yields derived above ($z_1$ until $z_{20}$ for the euro). The last forward rate pre and all forward rates post the FSP, if liquid maturities beyond the FSP exist, are used to calculate the LLFR.

### THE EXTRAPOLATED PART

First the LLFR is computed based on forward rates between the last liquid point before the FSP (15 years for the euro) and the FSP itself (20 years for the euro) and forward rates derived from liquid maturities according to the annual DLT assessment, where available, after the FSP (currently 25, 30, 40, and 50 years for the euro). In the following, rates are continuously compounded.

This means the following for the euro given the current DLT assessment of the swap rates,

$$LLFR = w_{20} * f_{15,20} + w_{25} * f_{20,25} + w_{30} * f_{20,30} + w_{40} * f_{20,40} + w_{50} * f_{20,50}$$

The weighting factors $w_x$ are based on the liquidity assessment of the swap market, where $V_x$ represents the annual average notional amount traded for a particular maturity point x:

$$w_{20} = \frac{V_{20}}{V_{20} + V_{25} + V_{30} + V_{40} + V_{50}}$$

Next, the forward rates beyond the FSP are then extrapolated according to the following formula:

$$f_{20,20+h} = \ln(1 + UFR) + \left(LLFR - Ln(1 + UFR)\right) * B(a,h)$$

$$B(a,h) = \frac{1 - e^{-ah}}{ah}$$

where h takes on values from 1 to the desired maturity beyond the FSP and *a* is the convergence factor and is equal to 10%.

The convergence factor plays a role in the extrapolation of the post-FSP forward rates. The convergence factor determines the speed of the post-FSP convergence to the UFR. The greater the convergence factor, the faster the extrapolated forward rates will converge to the designated UFR. Compared to the current method, the speed of convergence and the criteria to reach the UFR in 40 years after the LLP within three basis points are not used anymore.

In 2019, the Dutch body for pension funds supervision adopted a proposal to set the convergence factor to 2% based on recent data used in two versions of the Vasicek model and backed by additional scientific evidence.[15] In the proposed method, the factor is set to 10%. This was chosen out of conservatism, given the big impact of a larger change, and as a step toward using more market-consistent data. The size of this parameter could be reassessed and recalibrated in future reviews.

Finally, post-FSP zero coupon rates are extrapolated as follows.

$$z_{20+h} = exp\left(\frac{20 * z_{20} + h * f_{20,20+h}}{20 + h}\right) - 1$$

## Applying the volatility adjustment

The application of the volatility adjustment is similar to the current Smith-Wilson extrapolation method. However, rather than extrapolating, again, the basic risk-free term structure after adding the VA, the VA is added to the forward rates. For the rates up to the FSP, this is done in the following way:

$$f^{VA}_{x,x+y} = f_{x,x+y} + VA$$

The VA is also added to the last liquid forward rate, LLFR, the rate from which the extrapolation starts at the FSP, but only to the last forward rate before this FSP. For the euro this implies the following:

$$LLFR^{VA} = w_{20} * f^{VA}_{15,20} + w_{25} * f_{20,25} + w_{30} * f_{20,30} + w_{40} * f_{20,40} + w_{50} * f_{20,50}$$

[15] See for more information and details the official document, in Dutch. https://www.rijksoverheid.nl/binaries/rijksoverheid/documenten/kamerstukken/2019/06/11/advies-commissie-parameters/advies-commissie-parameters.pdf

This is similar to the current Smith-Wilson method, where the VA is also added to the rate for the last liquid point from which the extrapolation starts. The level of the UFR is not adjusted in the alternative extrapolation method, same as for the current Smith-Wilson method.

# Hedging implications

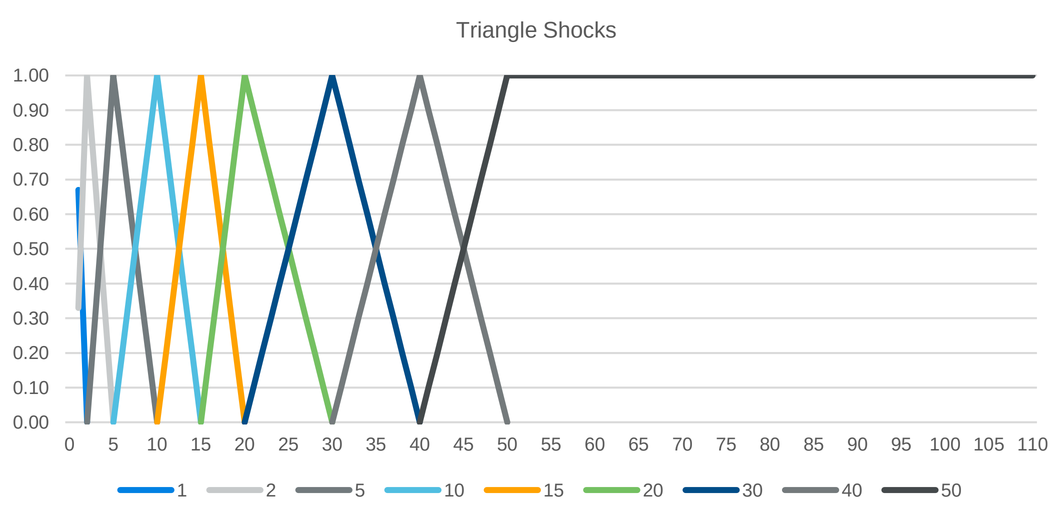
Now that we understand the background and technicalities of the extrapolation curves, we discuss the implications for hedging strategies. Here we focus on different approaches to measure key rate sensitivities, discuss the impact of hedge objectives, and introduce different approaches to measuring key rate sensitivities.

There are multiple approaches that can be used to calculate key rate duration (or rho), depending on the goal of the modeler. Some methodologies are also more sensitive to the underlying model used to construct the yield curve. Some of the more common approaches include:

1. Perturbing the par curve: Creation of bumps. In bumping, we form new curves indexed by i: To create the $i^{th}$ curve, one bumps up the $i^{th}$ input rate by, say, one basis point, and bootstraps the curve again.
2. Perturbing the forward curve with triangles. One approach is to again form new curves, again indexed by i: The $i^{th}$ curve has the original forward curve incremented by a triangle, with left hand endpoint at $t_{i-1}$, fixed height say one basis point and apex at $t_i$, and right hand endpoint at $t_{i+1}$. (The first and last triangle will in fact be right angled, with their apex at the first and last time points, respectively.)
3. Perturbing the forward curve with boxes. In boxes: The $i^{th}$ curve has the original forward curve incremented by a rectangle, with left hand endpoint at $t_{i-1}$ and right-hand endpoint at $t_i$, and fixed height say one basis point. Such a perturbation curve corresponds exactly with what we get from bumping, if one of the inputs is a $t_{i-1} \times t_i$.
4. FRA rate: We bump this rate, and we use the raw interpolation method.
5. Perturbing the par curve with triangles. This takes the approach in 2, but applies it to adjacent instruments, with the slope of the triangles aligned with the span between each instrument, which shapes how the points are interpolated from the shock of each instrument.

Figure 1 illustrates the construction of triangle shocks, whether on par rates or forward rates.

**FIGURE 1: TRIANGLE SHOCKS**



Figures 2 to 5 show the full spot curves for both the SW and AM approaches. We also graph the parallel up and down 10 bps shocks on the par rates, without shocking the FSP. That way, we have visual conception for how the yield curves look and respond to movements in the underlying par rates.

**FIGURE 2: SMITH-WILSON**

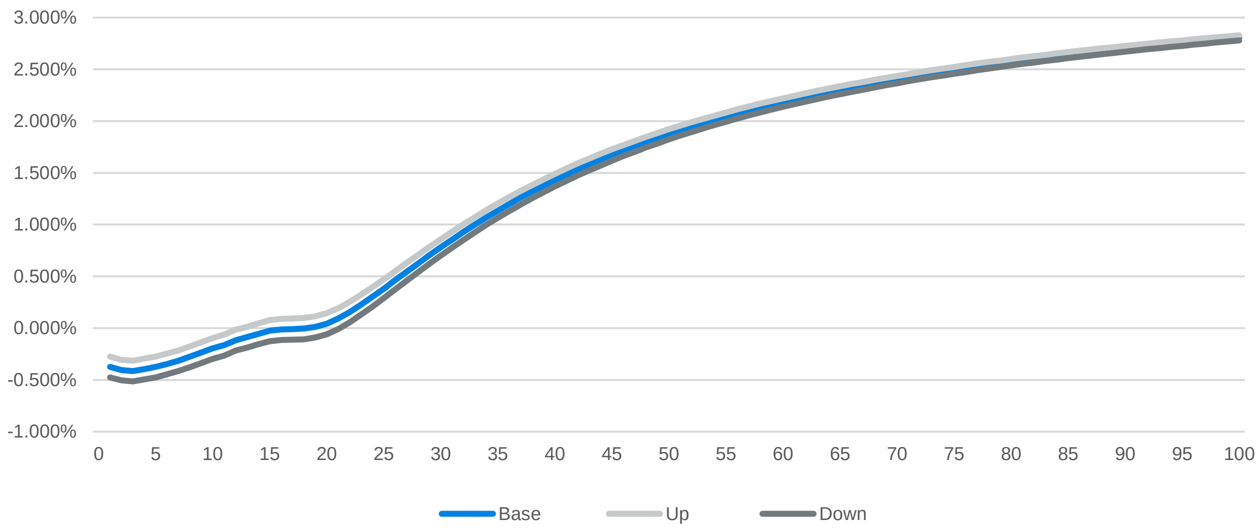



**FIGURE 3: SMITH-WILSON SHOCK - BASE**

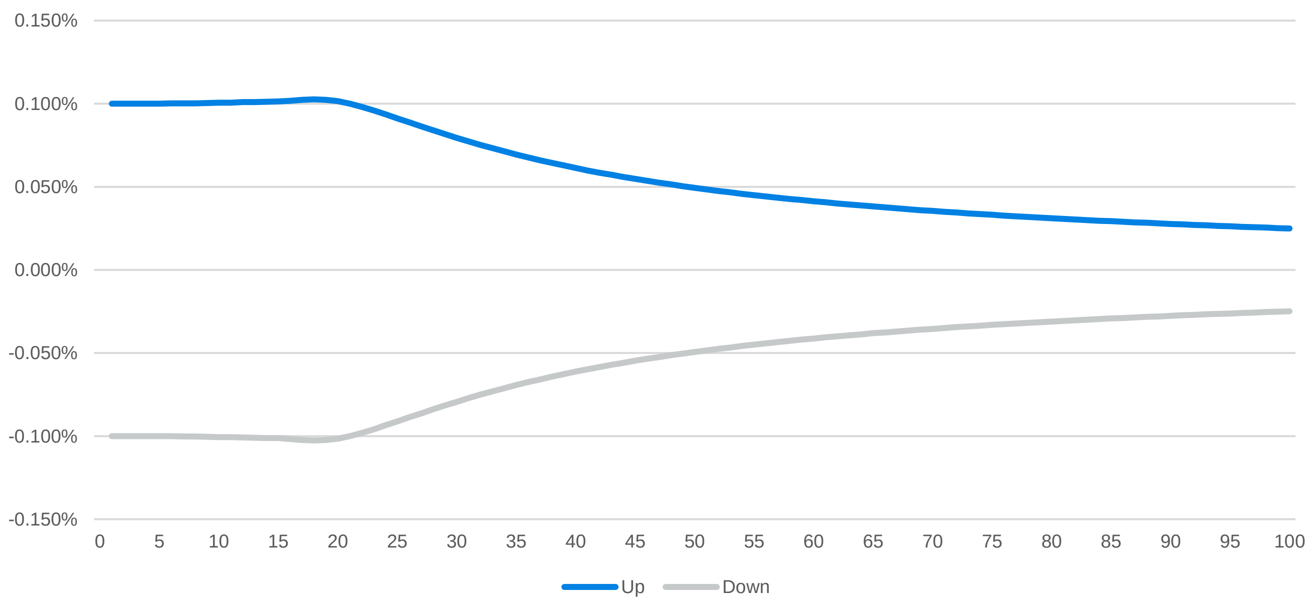


FIGURE 4: ALTERNATIVE METHODOLOGY

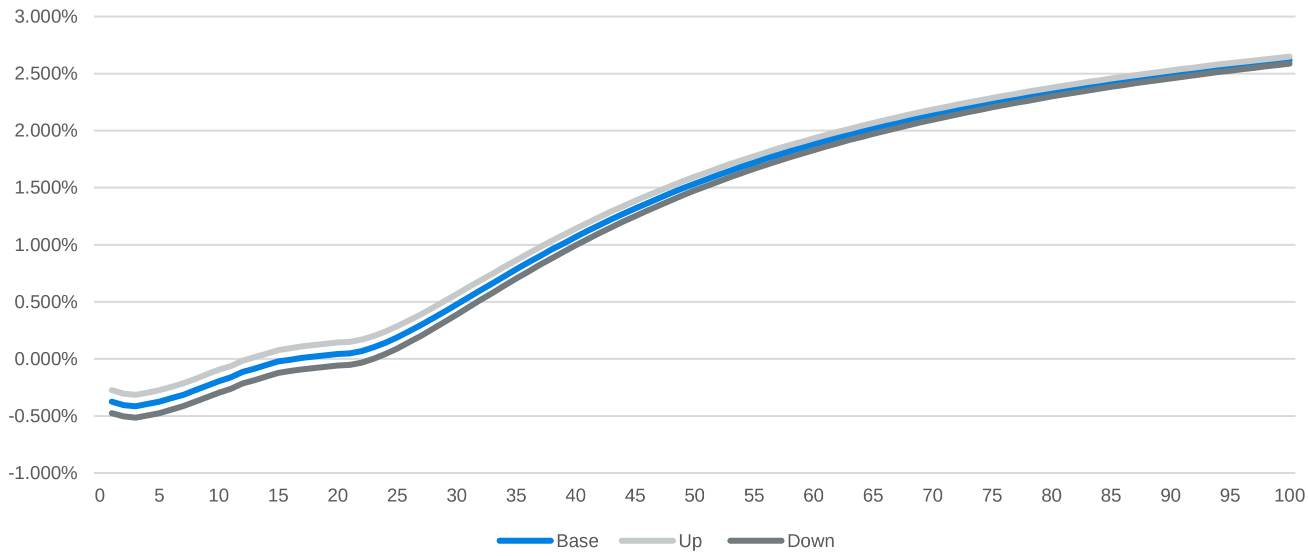
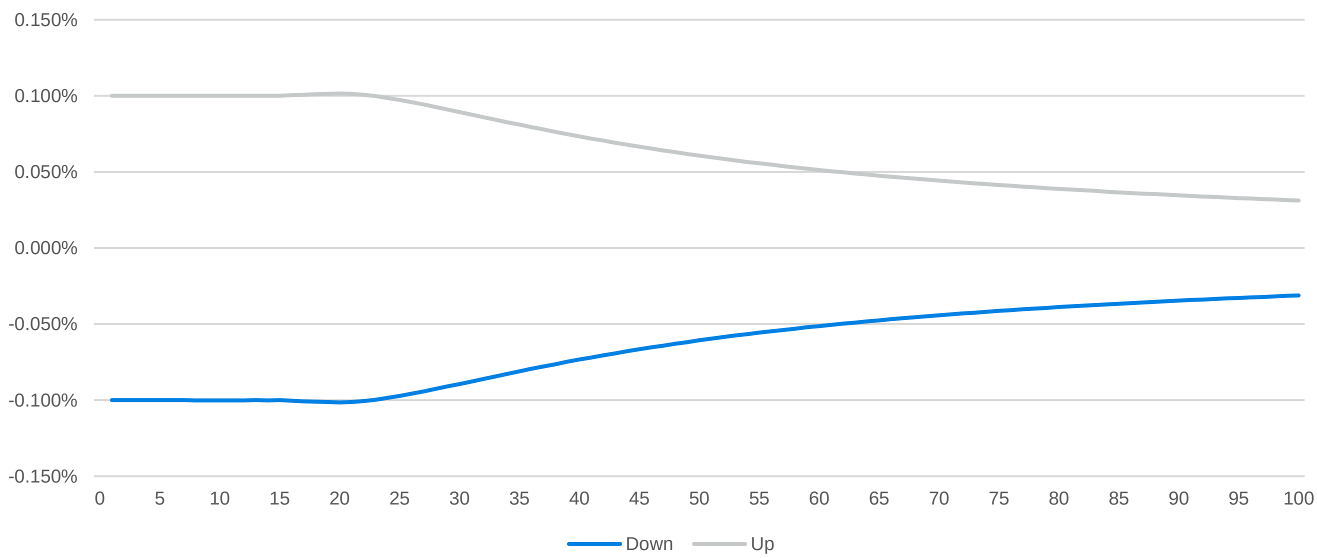



FIGURE 5: ALTERNATIVE METHOD SHOCK - BASE




Figures 4 and 5 visually illustrate the main feature of the alternative method, when parametrized as EIOPA recommends, relative to the SW: The yield curve has less curvature and the long-term rate is slightly lower for the AM. The AM still contains the well-known bumpiness of the SW approach, but it is more muted. Other than that, they exhibit very similar behaviors in response to parallel shocks.

## PAR VERSUS FORWARD SENSITIVITIES

An important part of the hedge strategy is the definition of the hedge objective. In this case, we do not refer to the decision to stabilize own funds or stabilize the Solvency II ratio, but on a more technical decisions: Base your hedge strategy on par or forward rates.

The original calibration for the determination of the discount curves uses par rates as an input. It is natural, then, for hedgers to use par rates to calculate market sensitivities, with concomitant benefit that this will likely also minimize volatility in reporting hedge results. However, if a particular interpolation/extrapolation method gives rise to non-local behavior, then it becomes less clear whether par rates are the proper lens through which to set hedge targets. This issue does not manifest if forward rates are shocked, because the forward curve follows from the interpolated/extrapolated spot or par curve. Importantly, forward rates are also, by construction, local in nature. While there are many compelling arguments in favor of shocking forward rates, we will focus on only one limitation: shocking forward rates that coincide with the extrapolated portion of the curve.

In general, shocking forward rates after the LLP give rise to sensitivities that either do not have a matching asset with which to hedge, or allocate all exposure past the LLP to the end of the market data-derived portion of the curve. In either approach, you are no longer measuring a true economic sensitivity but a synthetic one. To avoid this, hedgers will generally only manage the portion of the curve that is actually market sensitive. Best practices mandate some additional measurement of the full economic sensitivity, even if it does not align with hedgeable instruments, because of the real risk that will emerge over time as the business ages and hedge rebalancing occurs. Thus, the hedge target will deviate from the full economic exposure. Risk managers should monitor this so that they can track expected hedge costs.

Hedgers frequently calculate sensitivities using forward rates, in part because their very nature make them somewhat immune to certain pathologies that curve construction methodologies can create. However, forward rates are predominantly derived values, and measuring sensitivities on them necessitates an extra step to determine matching hedges. Some hedgers eliminate this extra step by shocking the par rates directly and then deriving the yield curve consistent with the new, shocked par rates. When done one input at a time, this allows the hedger to determine exactly how much of that instrument to purchase to hedge the exposure.

Forward rate curves also are great at revealing the warts of any yield curve construction methodology, since forward curves do not undergo the same smoothing that tends to occur with par/spot curves. Below are plotted the forward curves, plus the parallel 10 bps shocks up and down, for both the Smith-Wilson and alternative methods. As expected, since the shocks were on the forward rates, both the up and down curves are exactly 10 bps above and below the base curve, respectively.

FIGURE 6: SMITH-WILSON FORWARD RATES

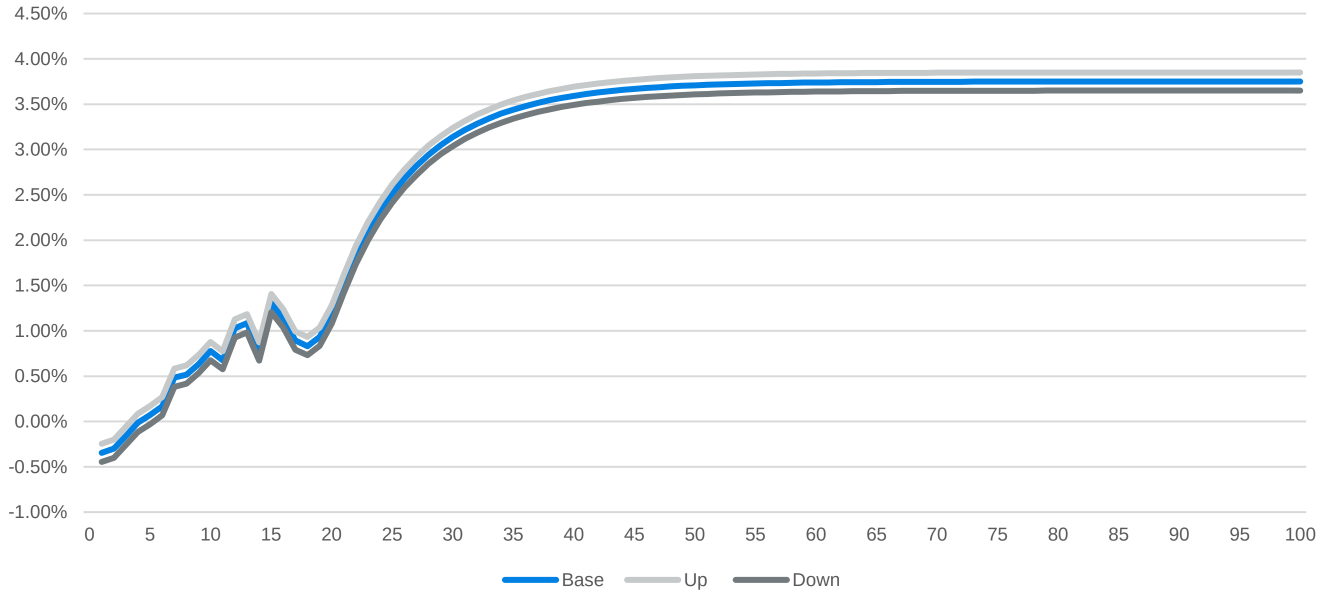
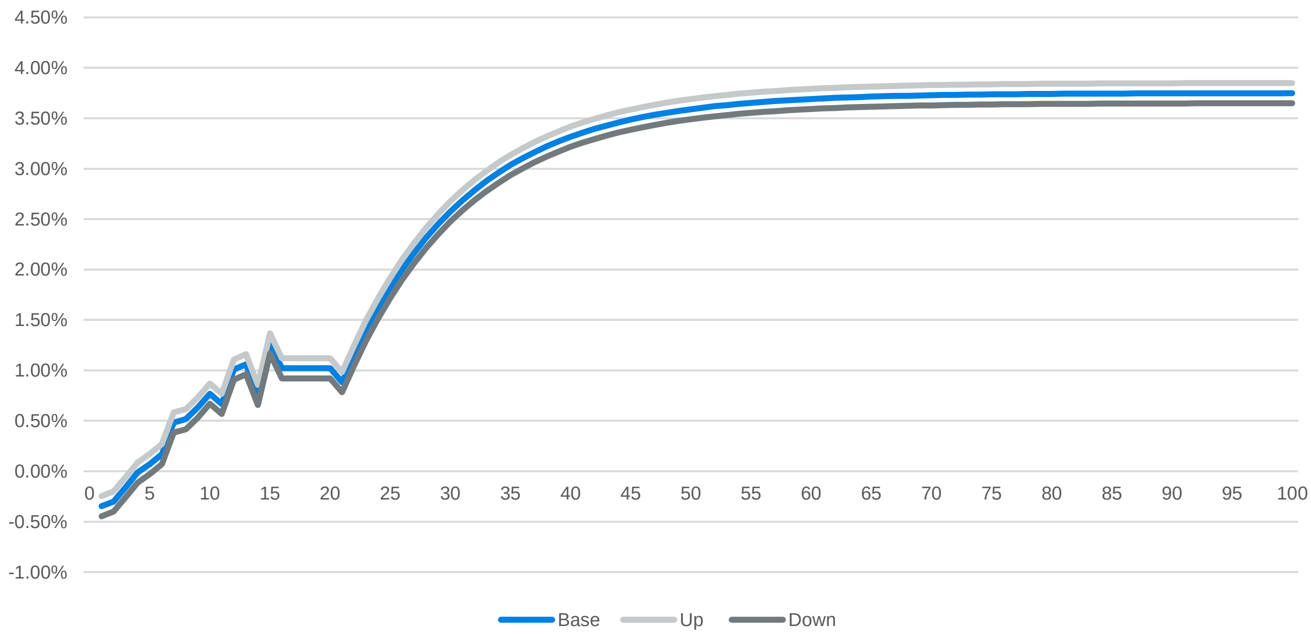



FIGURE 7: ALTERNATIVE METHODOLOGY FORWARD RATES




In Figures 6 and 7, there are obvious pathologies created by the interpolation algorithms, though they are somewhat more subdued under the alternative methodology. Any attempt at smoothing spot rates invariably creates less realistic forward rates, though some approaches (i.e., monotone convex) do a better job of managing the distortions.[16]

[16] Hagan & West (2006).

Another approach to evaluate the performance of the SW approach against the AM is to examine the non-local impact of shocks to key par rates. Below are charts that plot base curves against a parallel up shock and key rate shocks for years 1, 2, 5, 10, 15, 20, 30, 40, and 50. Each curve represents the impact on an up shock to each of those key rates, leaving all others fixed, and then interpolating and extrapolating the full curve consistent with that single shocked value. Figures 8 show the behavior of the Smith-Wilson method, and Figures 9 the alternative method. In each of these cases, we look at the change relative to base to better draw out the differences between the two approaches, which is less apparent if we are looking at the curves directly.

**FIGURE 8: SMITH-WILSON SHOCK - BASE**

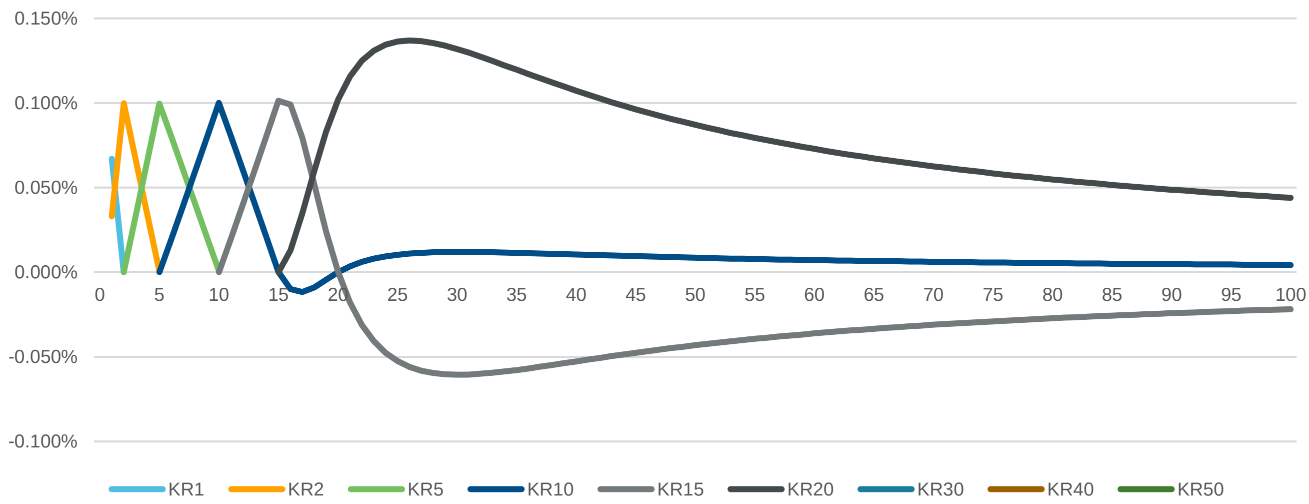
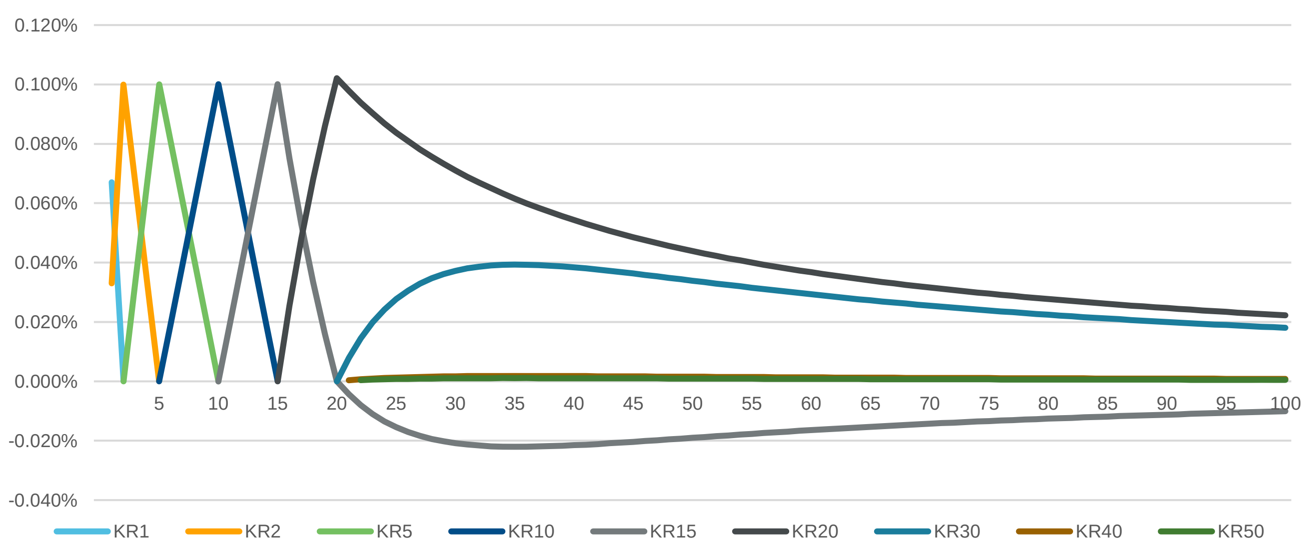



**FIGURE 9: ALTERNATIVE METHODOLOGY SHOCK - BASE**




While all figures reveal some amount of non-locality, in that there are notable changes for most of the curves at substantial distances from the shocked key rate, the SW curve shows greater instabilities. The most pronounced of these instabilities, which resemble squiggles, occur at key rates 5 through 15. The KR15 curve also has the most visible distortions, which results in an inversion under the SW approach (that is, the curve is not monotonically non-decreasing).

For completeness, we can also examine the resulting impact of using key forward rate shocks instead of key par rate shocks. In this case, for comparability, we will still look at the deltas (change from base**) relative to the base spot curve** (not forward), to illustrate the resulting perturbations on the entire yield curve. However, we stress that hedgers would not look to changes in the spot curve in this way, since the point of converting to forwards and shocking those is to hedge the forward rates, not the spot rates constructed therefrom.

**FIGURE 10: SMITH-WILSON FORWARD SHOCK-BASE**

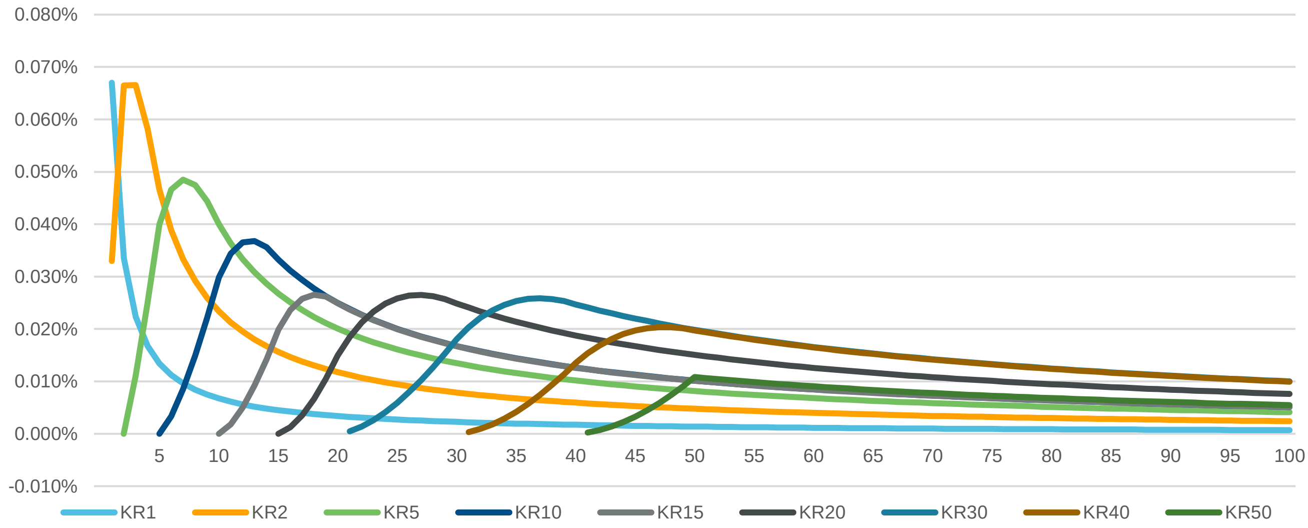
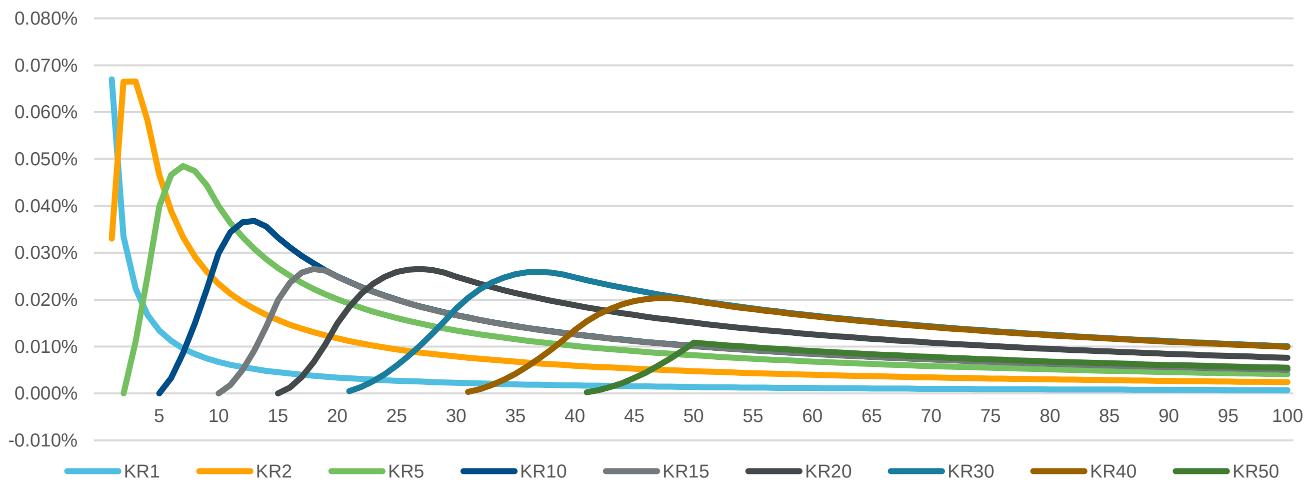



**FIGURE 11: ALTERNATIVE METHOD FORWARD SHOCK - BASE**




One of the more pleasant features of working in the forward rate space is less sensitivity to different methodologies, as is evident here, where the curves are essentially identical. Thus, there is much greater comparability and stability to changing methodologies, which is advantageous to hedgers operating in an industry where underlying methodologies are exogenously influenced by regulators, auditors, and other stakeholders, or endogenously through assumption or model updates internal to an organization.

One challenge of operating in forward space instead of par space is that the forward shocks look bizarre when the forward shocks are viewed on the spot curves. As is apparent in both curves, the initial key rate forward shocks create similar peaks to what we saw with the par shocks. However, each subsequent shock shows a lower peak (though not monotonically because of the role the extrapolation plays after year 20), but also long right tails, the fatness of which tends to increase as well. This is because forward rate shocks influence the spot rates at each point after the shocked key rate.

Finally, we can see the impact of these curves on DV01 and key rate DV01 (both par and forward) for an illustrative liability.[17]

**FIGURE 12: SW FORWARD KR DV01 (RELATIVE CHANGE)**

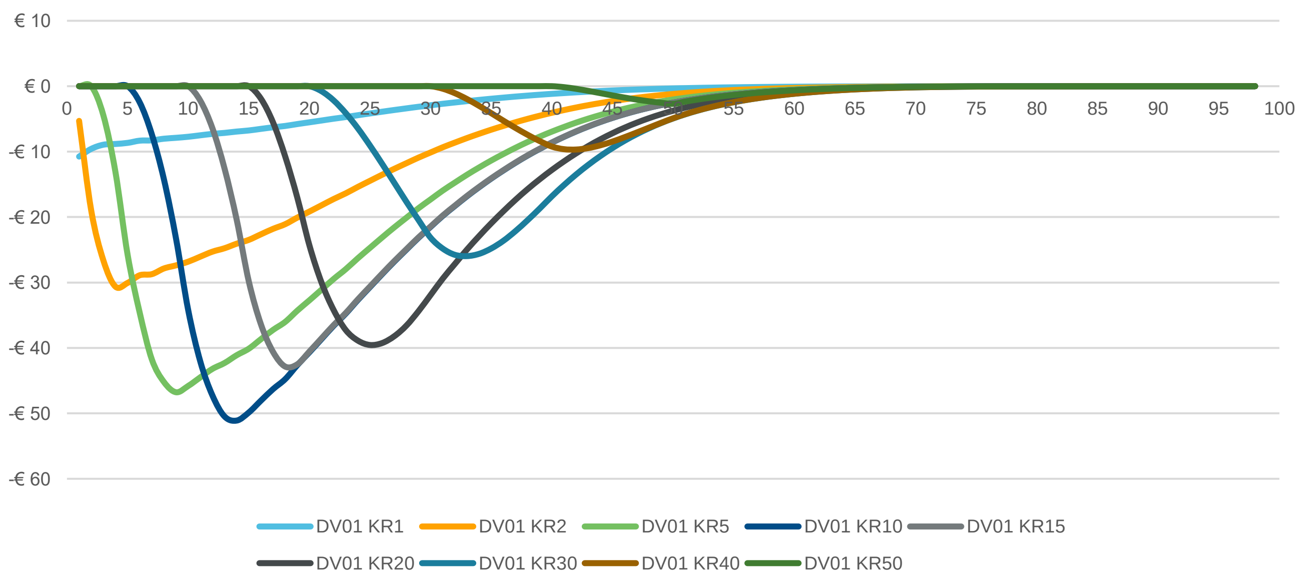
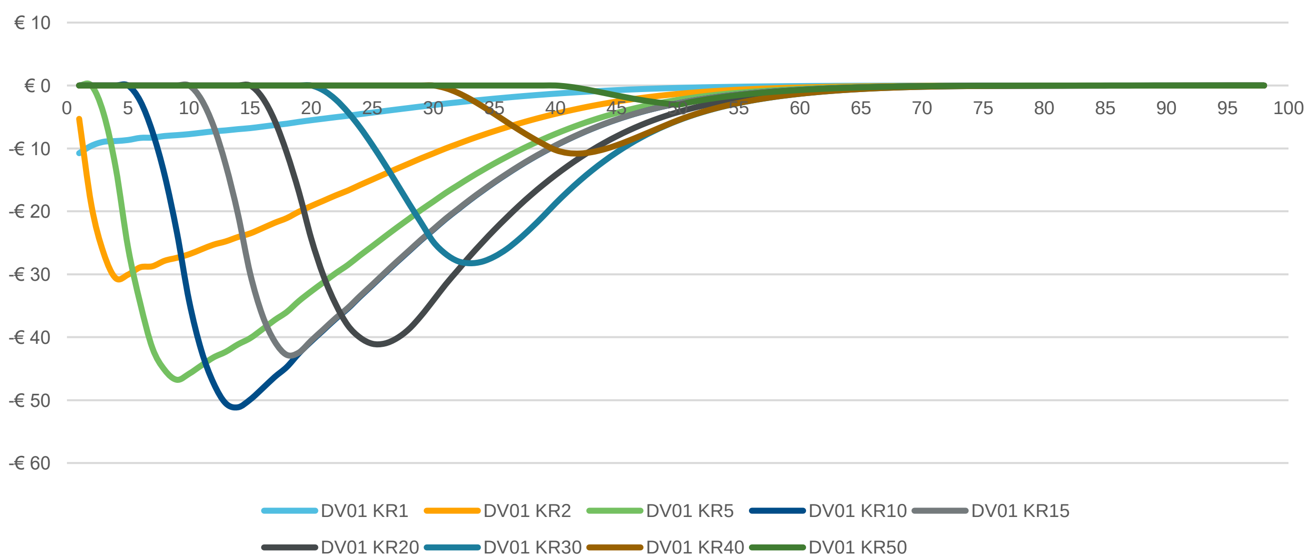



**FIGURE 13: AM FORWARD KR DV01 (RELATIVE CHANGE)**




[17] This liability does not reflect any actual modelled product, but a dummy product with exposure to the entirety of the yield curve. It is meant to be illustrative, not prescriptive. For the purposes of this illustration, we assumed hedge ratios equal to 100% for the 0 to 5-year bucket, based on a flat shock, and then 80% thereafter.

FIGURE 14: SW PAR KR DV01 (RELATIVE CHANGE)

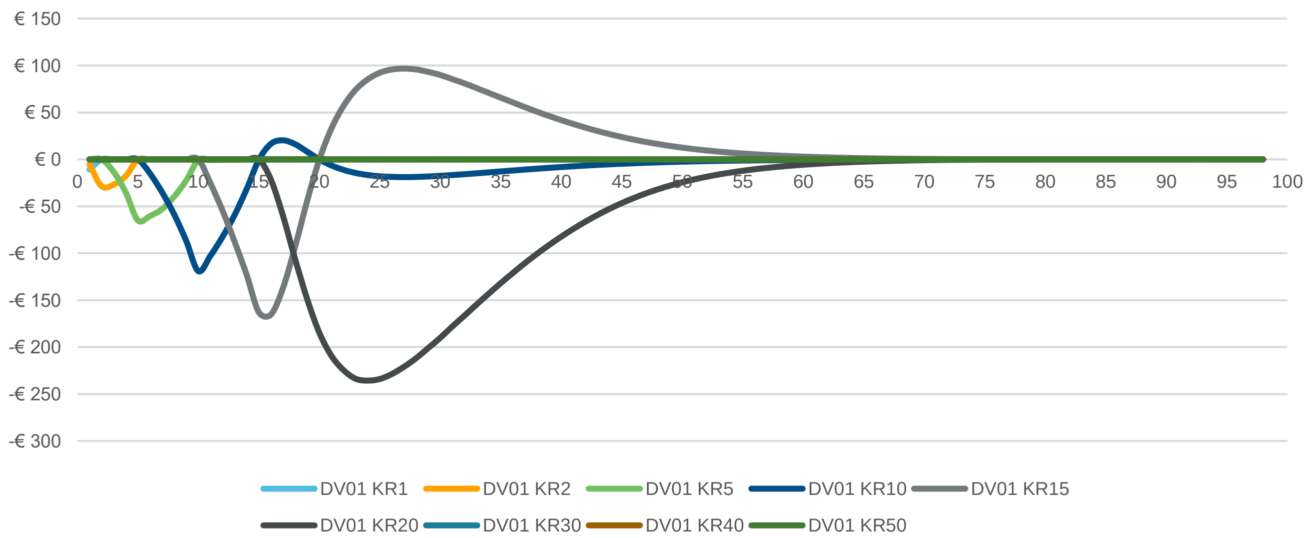
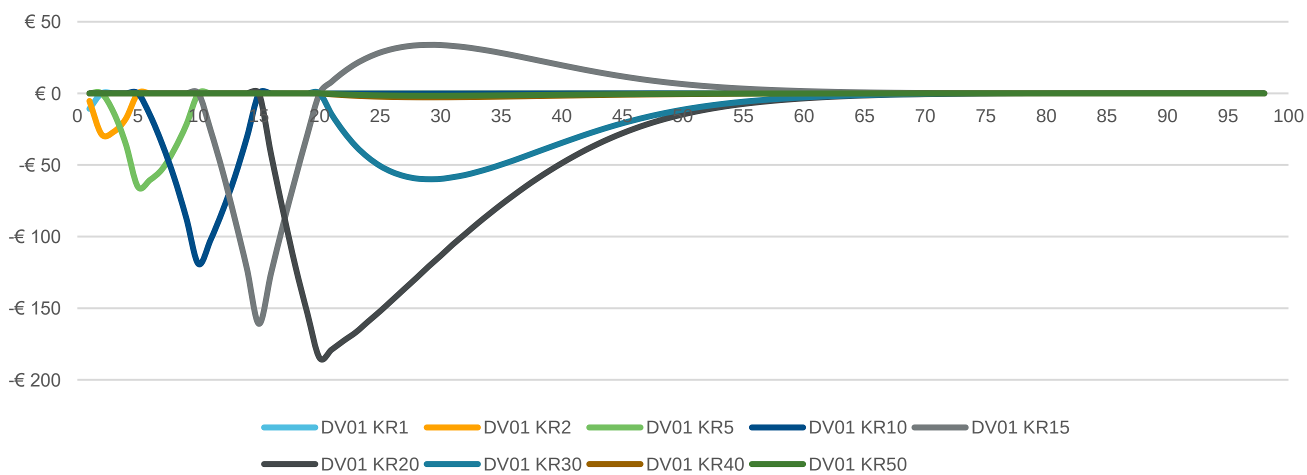



FIGURE 15: AM PAR DV01 (RELATIVE CHANGE)




These illustrations help show the AM does a bit of a better job smoothing out the shocks, and maintaining a bit more of the sharpness of the triangle shocks than under the Smith-Wilson, which creates a bit more noise. This, if anything, should help hedgers more effectively manage hedge program targets, even if they are hedging par rates.

However, all of these shocks do show both positive and negative variations, which reflect how the Par DV01 behaves under shocks relative to forward DV01. That is, when shocking par rates, you are invariably susceptible to the interpolation/extrapolation methodology. Attempts at smoothing the yield curve interject behaviors into the shocks that create waves patterns and greater non-locality (smoothing requires referencing points removed from the shocked point, which thereby allows the shocked point to influence the value of other referenced points in the interpolation/extrapolation).

Hedgers need not tie themselves to the alternative methodology, in the same way that not all hedgers target the Smith-Wilson-generated yield curve today. There are trade-offs for using a different yield curve. In particular, there will be a disconnect between changes in SRC and hedge effectiveness, but those trade-offs are quantifiable. Insurers can communicate the difference through an adjusted hedge effectiveness, for example. Another possibility is adjusting the parameters of the alternative methodology to make the curve closer to market consistent.

# Conclusions

This paper addressed quite a few issues related to EIOPA's new alternative methodology for yield curve construction. We started by providing a detailed discussion of how the AM approach differs from the current Smith-Wilson approach, how that methodology can be implemented, and then the potential implications of the change to hedgers. The goal was introductory yet also demonstrative for how the new approach will affect hedging insurance liabilities.

The goal of the Solvency II review was not to improve hedge results but to address the tension that exists between being fully market consistent, yet avoiding adoption of features that promote too much pro-cyclicality in SCR. The alternative method is a reasonable step in that direction. From a hedging perspective, the move also alleviates, but does not cure, some of the concerns with the Smith-Wilson approach. Most significantly, the AM mutes distortions caused by the SW, especially around the 15-year point, even if one shocks par rates to estimate key rates. There remain features in the curve that will continue to make hedging a more challenging task (most significantly, the waves that can give rise to alternating long and short positions), but the new methodology also allows for greater flexibility in building and managing hedge programs with targets that lie along the spectrum from Solvency II to pure market consistent.

To that last point, what has remained unaddressed is the role played by the various calibration parameters, including α, the FSP, and the UFR. Through careful management, insurers can modify these parameters to modify hedge targets in accordance with shorter-term or longer-term economic outlooks, which can allow for a more hybrid economic/Solvency II hedge target. Space constraints prevent substantive discussion of this subject here, but we believe it a fruitful avenue for future research and one we intend to pick up in a later paper.

### CITATIONS

European Insurance and Occupational Pensions Authority. (2010). QIS5 Risk-Free Interest Rates – Extrapolation Method, QIS5 Background Document. EIOPA.

European Insurance and Occupational Pensions Authority. (2020). Consultation Paper on the Opinion on the 2020 review of Solvency II. EIOPA.

European Systemic Risk Board. (2017). Regulatory risk-free yield curve properties and macroprudential consequence. ESRB.Hagan, P., & West, G. (2006). Interpolation Methods for Curve Construction. Applied Mathematical Finance, 13(2), 89-129. https://doi.org/10.1080/13504860500396032

Lagerås, A., & Lindholm, M. (2016). Issues with the Smith–Wilson method. Insurance: Mathematics And Economics, 71, 93-102. https://doi.org/10.1016/j.insmatheco.2016.08.009



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